

**Comparative Effectiveness Research
Methods Training**

Module 4: Propensity Score Application

J. Michael Oakes, PhD
Associate Professor
Division of Epidemiology
University of Minnesota
oakes007@umn.edu

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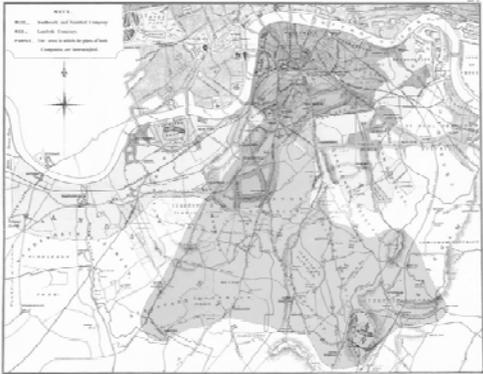
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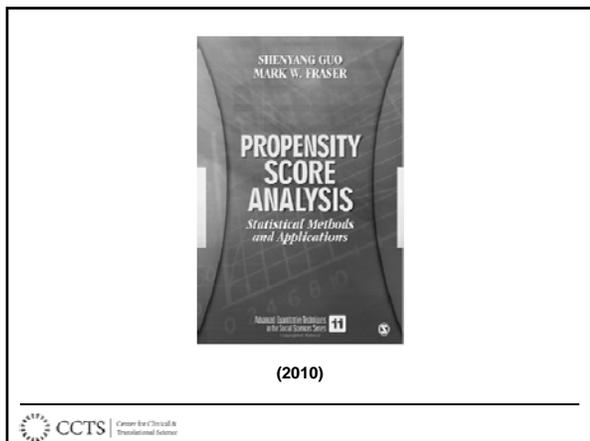
A little (more) about me.



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Module #4 Outline

1. Review of Core Ideas
2. ATE, ACE, ATT, TOT
3. A Typical Analysis
4. Propensity Score Methods
5. Issues & Assumptions
6. Review
7. Questions

1. Review Core Ideas



Potential Outcomes

Condition Assigned	Outcome if Treated, Y^1	Outcome if not Treated, Y^0
Treatment	Observed	Unobservable Counterfactual
Control	Unobservable Counterfactual	Observed



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Analysis of Experimental Data

$$Y = \alpha + \beta_1 T + \varepsilon$$

$\hat{\beta}_1 \Rightarrow \bar{\Delta} = \text{average causal effect}$

T is (0,1) treatment indicator which, for large samples, is independent of background characteristics by study design (ie, randomization)



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Absent Randomization

$$Y = \alpha + \beta_1 T + \beta Z + \varepsilon$$

Covariates, Z, serve to adjust groups for confounding...



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Absent Randomization

Unless specification of the model, including X, is perfect, bias results

$$\beta_1 = \bar{\Delta} + \text{BIAS}$$



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Propensity Score Methods

An approach to confounder control that better mimics the experimental approach.

Introduced by Rosenbaum and Rubin in 1983



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Propensity Score, $p(z)$

In the analysis of treatment effects, suppose that we have a binary treatment T , an outcome Y , and background variables Z . The propensity score is defined as the conditional probability of treatment given background variables:

$$p(z) \equiv \Pr(T = 1 | Z = z)$$

↑
Propensity score



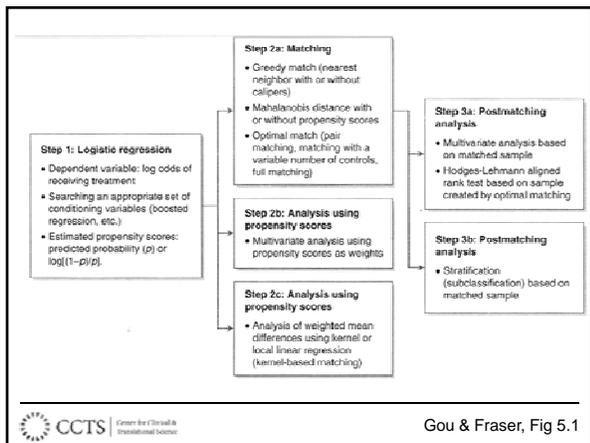
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Ignorable TAM

$$TAM \perp Y(0), Y(1) \mid p(z)$$

↑
Propensity score





Because they require us to think about the ideal experiment we would have liked to have conducted, propensity score methods are a better tool than multiple regression. Setting aside the outcome variable, Y , until it's time to assess differences between observed outcomes and counterfactual substitutes, is an invaluable addition to the practice of applied research.



2. ATE, ACE, ATT, TOT

What "effect" do we wish to estimate?

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Ideally, we'd like the treatment effect for each individual in our study. If we could observe every person and their counterfactual we could just take the average across all persons as an estimate of delta.

$$\tau_i = Y_i(1) - Y_i(0)$$
$$\bar{\tau} \Rightarrow \Delta$$

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But of course we cannot calculate a causal effect for a particular person.

We must move up the population level.

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The **average treatment effect (ATE)** is the difference in the average of the outcome variable in the treatment group minus the average of the outcome variable in the control group. ATE is the same as the **average causal effect (ACE)**.

$$ATE = ACE = E[Y(1) - Y(0)]$$



The **average treatment effect (ATE)** is the difference in the average of the outcome variable in the treatment group minus the average of the outcome in the control group. ATE is the same as the **average causal effect (ACE)**.

$$ATE = ACE = E[Y(1) - Y(0)]$$

Often desired and easily estimable in RCTs



The **average treatment effect on the treated (ATT)** is the mean difference between those actually treated or exposed and their counterfactuals. ATT is the same as the **treatment effect on the treated (TOT)**.

$$ATT = TOT = E[Y(1) - Y(0) | T=1]$$



The **affect of the treatment on the treated (ATT)** is the mean effect of those actually treated or exposed. ATT is the same as the **treatment effect on the treated (TOT)**.

$$ATT = TOT = E[Y(1) - Y(0) | T=1]$$

Often better for observational designs



When we randomize the treatment assignment mechanism (TAM) is independent of the outcomes and all subjects have a positive probability of being treated. Accordingly, the $ATT = ATE$:

$$ATT = ATE = E(Y | T = 1) - E(Y | T = 0) = E[Y(1) - Y(0)]$$

This is because randomization produces exchangeable groups (ie, balance) and yields excellent counterfactual substitutes. In other words, the control group serves to substitute for the unobservable counterfactuals of the treatment group, at least with large samples.



But when we do not randomize the treatment assignment mechanism (TAM) is rarely independent of the outcomes and some subjects may have a zero probability of being treated. Accordingly,

$$ATT \neq ATE$$

Thus we are often better off comparing treated subjects to the best counterfactuals we can find for them. The best ones are the non-treated subjects that have the same probability of being treated as the treated subjects being studied (or the set of them). This theoretically satisfied the critical assumption about independence of the treatment assignment mechanism: $TAM \perp Y(0), Y(1) | p(z)$



Incredibly, neither of the desired effect estimates (ATT, ATE) is (easily) identifiable through regression modeling in an observational study.



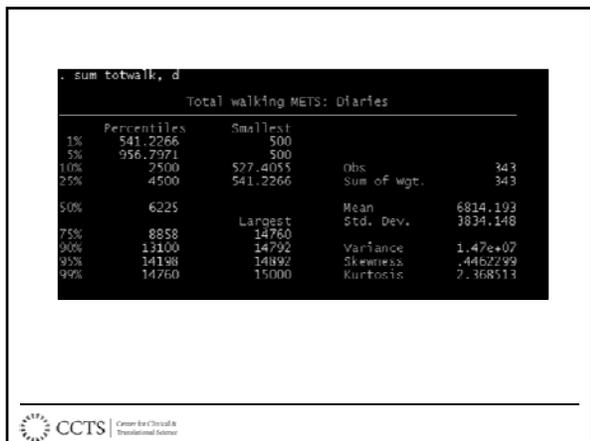
3. Typical Analysis

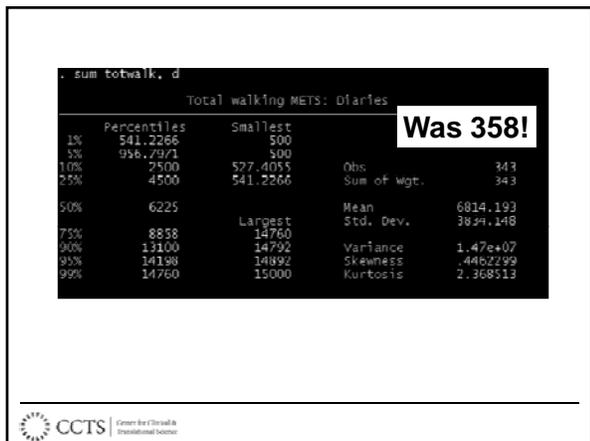


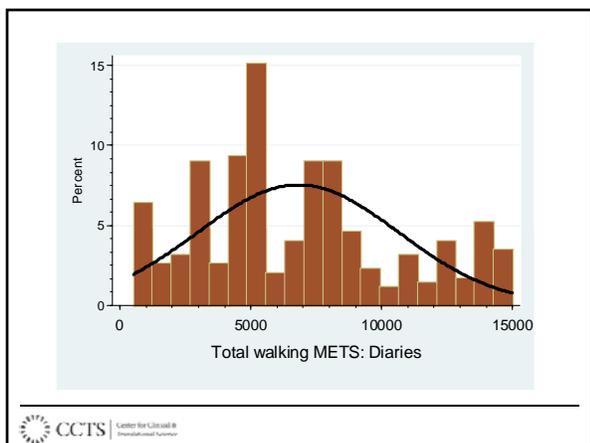
Let's use Stata

What is Stata? Stata is a full-featured statistical programming language for Windows, Macintosh, Unix and Linux. It can be considered a "stat package," like SAS, SPSS, or R.









```

. tab walkable

```

Fictitious Walkability Measure	Freq.	Percent	Cum.
No	170	47.49	47.49
Yes	188	52.51	100.00
Total	358	100.00	

```

. tab walkable, sum(totwalk)

```

Fictitious Walkability Measure	Mean	Std. Dev.	Freq.
No	4466.9781	2349.2146	164
Yes	8964.7132	3669.4117	179
Total	6814.1926	3834.1481	343

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t-test of totwalk (Y) by walkable (X)

```

. ttest totwalk, by(walkable) unequal

```

Two-sample t test with unequal variances

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf. Interval]
Tx	170	8964.713	274.2640	3669.414	8423.484 9505.942
Cx	164	4466.978	183.4428	2349.215	4104.747 4929.209
combined	343	6814.193	207.0245	3834.148	6406.991 7221.394
diff		4497.735	329.9583		3848.461 5147.009

diff = mean(Tx) - mean(Cx) t = 13.6312
 Ho: diff = 0 Satterthwaite's degrees of freedom = 306.007
 Pr(C < t) = 1.0000 Pr(T > |t|) = 0.0000 Pr(C > t) = 0.0000

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```

. ttest totwalk, by(walkable) unequal

```

Two-sample t test with unequal variances

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf. Interval]
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Bivariate Regression: totwalk (Y) on walkable (X)

```

- reg totwalk walkable /* Fit a crude model */
    
```

Source	SS	df	MS		
Model	1.7314e+09	1	1.7314e+09	Number of obs =	342
Residual	3.2963e+09	341	9666463.74	F(1, 341) =	179.11
Total	5.0276e+09	342	14700691.9	Prob > F =	0.0000
				R-squared =	0.3444
				Adj R-squared =	0.3274
				Root MSE =	3109.1

	coef.	std. err.	t	P> t	[95% conf. interval]
walkable	4497.735	336.0721	13.38	0.000	3836.7 5138.77
_cons	4466.978	242.7794	18.40	0.000	3989.444 4944.512

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```

- reg totwalk walkable /* Fit a crude model */
    
```

Source	SS	df	MS		
Model	1.7314e+09	1	1.7314e+09		
Residual	3.2963e+09	341	9666463.74		
Total	5.0276e+09	342	14700691.9		

	coef.	std. err.	t	P> t
walkable	4497.735	336.0721	13.38	0.000
_cons	4466.978	242.7794	18.40	0.000

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Assess differences in averages of suspected confounders across treatment (walkable) and control (no walkable) conditions.

Use a t-test
(better methods available)

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```

reg totwalk walkable age college hhinc, /* retain just stat signif */

```

Source	SS	df	MS			
Model	4.0057e+09	4	1.0014e+09	Number of obs =	349	
Residual	917549180	115	7774077.4	F(4, 335) =	367.62	
Total	4.9181e+09	339	14508139.9	Prob > F =	0.0000	
				R-squared =	0.8145	
				Adj R-squared =	0.8122	
				Root MSE =	1650.5	

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
walkable	2205.745	198.7595	11.10	0.000	1814.771 2596.719
age	-87.34888	3.305979	-26.70	0.000	-77.78433 -58.9134
college	2193.281	218.4568	10.42	0.000	1779.217 2807.344
hhinc	685.9745	33.20722	20.66	0.000	620.6535 751.2954
_cons	3338.407	351.1121	9.51	0.000	2647.744 4029.069

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```

. estimates table crude adjusted pvalue, b(%7.2f) keep(walkable) star

```

variable	crude	adjusted	pvalue
walkable	4497.74***	2215.29***	2205.75***

Legend: * p<0.05; ** p<0.01; *** p<0.001

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```

. estimates table crude adjusted pvalue, b(%7.2f) keep(walkable) star

```

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walkable	4497.74***	2215.29***	2205.75***

Legend: * p<0.05; ** p<0.01; *** p<0.001

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4. Propensity Score Analysis



Nearest Neighbor Matching within caliper

```
* First let's do nearest neighbor matching w/in caliper and check balance
. psmatch2 walkable age tenure male white married college hhinc, ///
  logit neighbor(1) common caliper(0.1) time

Logistic regression      Number of obs =      49
                        LR chi2(7) =      78.96
                        Prob > chi2 =      0.0000
                        Pseudo R2 =      0.1652

Log likelihood = -199.47867
```

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
walkable						
age	-.0329125	.0074342	-4.43	0.000	-.0474833	-.0183418
tenure	-.0294465	.0111162	-1.93	0.054	-.0503382	-.0085548
male	-.0416152	.0700094	-1.09	0.276	-.1133372	.0301067
white	-.1120942	.3194757	-0.35	0.726	-.738163	.5141566
married	.4441075	.2821876	1.57	0.116	.0973777	.7908373
college	1.484274	.7791028	1.91	0.060	.0440903	2.9244577
hhinc	.1099414	.0538205	2.04	0.041	.0044551	.2154277
_cons	-.0781083	.5615065	0.14	0.889	-1.027542	1.178759

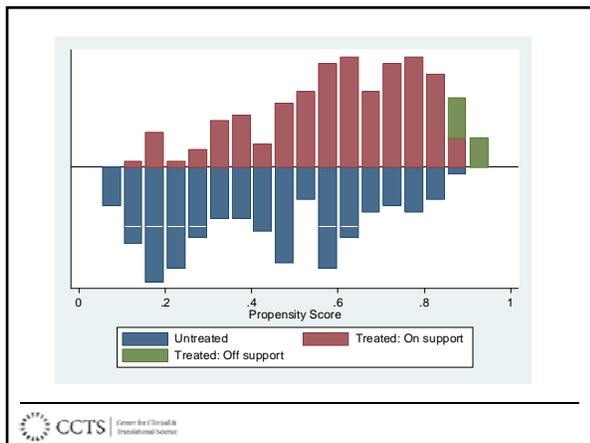
Assess Balance

```
. postest age tenure male white married college hhinc /* Evaluate balance */
```

Variable	Sample	Mean Treated	Mean Control	%Bias	%Reduct. (Bias)	L-Test t	p> t
age	Unmatched	44.309	52.413	-47.8		-4.47	0.000
	Matched	46.574	46.636	-0.4	99.2	-0.03	0.972
tenure	Unmatched	14.119	13.068	8.8		0.82	0.413
	Matched	13.733	13.845	0.9	89.4	0.09	0.928
male	Unmatched	.3427	.32335	4.1		0.38	0.704
	Matched	.34568	.32222	26.1	-538.2	2.40	0.014
white	Unmatched	.80899	.68892	27.9		2.60	0.010
	Matched	.79014	.61491	-9.7	79.5	-0.39	0.378
married	Unmatched	.29775	.49105	-40.2		-3.74	0.000
	Matched	.32736	.32099	1.3	96.8	0.32	0.906
college	Unmatched	.82022	.48503	75.0		6.99	0.000
	Matched	.80747	.74074	13.8	81.6	1.32	0.187
hhinc	Unmatched	6.6798	5.1976	53.3		4.94	0.000
	Matched	6.3765	6.8642	-17.5	67.1	-1.59	0.114

psmatch2: Treatment assignment	psmatch2: Common support		Total
	Off support	On support	
Untreated	0	162	162
Treated	12	164	176
Total	12	326	338

There are no matches for 12 of the "treated" subjects



Bootstrap, k=1000

```

. bootstrap psmatch2 malmale age male white married collgeo hhinc, outcomelotwalk
(5 0)
Command: psmatch2 malmale age male white married collgeo hhinc, outcomelotwalk
Statistic: _bc_1 = pCoff_forwalk
Note: Table generated via Stata's nlcom.

Bootstrap statistics      Number of obs   =   338
                          replications   = 10000

Variable      Reps  Observed   Bias  Std. Err.  [95% Conf. Interval]
-----
_bc_1         10000  2513.21  156.1548  284.1708   1055.894   3070.536  (N)
              10000  2130.834  3257.353  1875.054   2922.027  (P)
              10000  1875.054  2922.027  (bc)
    
```

Note: N = normal
P = percentile
bc = bias-corrected

```

bootstrap psmatch2 walkable age male white married college hhsinc, outcome(totwalk)
* (0)
Command: psmatch2 walkable age male white married college hhsinc, outcome(totwalk)
Statistic: _bs_1 = e(ata_0totwalk)
Note: label truncated to 80 characters

Bootstrap statistics      Number of obs   =   399
                        Replications   =  10000

Variable      Repls  Observed      Bias  Std. Err.  [95% Conf. Interval]
-----
_bs_1         1.0e+04  2513.21  156.1548  284.3208   1955.884   3070.536  (0)
                                     2130.879   2627.253  (P)
                                     1875.654   2922.027  (BC)

Note: N = normal
      P = percentile
      BC = bias-corrected
    
```



```

Bootstrap statistics      Number of
                        Replicati
Variable      Repls  Observed      Bias  Std. Err.  [95% Co
-----
_bs_1         1.0e+04  2513.21  156.1548  284.3208   1955
                                     2130.8
                                     1875.0

Note: N = normal
      P = percentile
      BC = bias-corrected
    
```



```

list _treated _support totwalk _totwalk diff
-----
1.  Untreated  On support  7693      .      .
2.  Untreated  On support  7340.5    .      .
3.  Untreated  On support  785.1813  .      .
4.  Treated   On support  4808     6710.1045  1147.896
5.  Untreated  On support  4500     .      .
6.  Treated   On support  8235     1202     7013
7.  Untreated  On support  2544.453  .      .
8.  Treated   On support  13611.5  8104.8281  5506.672
9.  Untreated  On support  912.5    .      .
10. Treated   On support  10647.59  6061.793  4585.788
11. Treated   On support  1098     4608     400
12. Treated   On support  1043.5    .      .
13. Treated   On support  7418.602  4608     2720.602
14. Treated   On support  9100.874  8039.0894  1201.783
15. Treated   On support  8090     6803.4849  1892.515
16. Treated   On support  8438.125  1094     3164.125
17. Treated   On support  11709.32  6996     4011.322
18. Untreated  On support  4838.504  .      .
19. Untreated  On support  4934.194  6803.4849  -1869.291
20. Treated   On support  8699.115  7247.5    1451.615
21. Treated   On support  1900     4098     -1798
22. Untreated  On support  4549.5    .      .
23. Treated   On support  10200    8288.8515  1901.146
24. Untreated  On support  8099.089  .      .
25. Untreated  On support  3771.216  6681     1101
    
```



Rosenbaum Bounds

A method of sensitivity analysis that uses a parameter "gamma" to measure the degree of departure from random assignment of treatment.

Imagine that 2 subjects with the same observed characteristics differ in the odds of receiving the treatment by at most a factor of "gamma".

In a randomized trial, randomization of the treatment ensures that gamma = 1. In an observational study, if gamma = 2 and two subjects are identical on matched covariates then one might be twice as likely as the other to receive the treatment because they differ in terms of an unobserved covariate.



```

rbounds diff, gamma(1(17)) /* Rosenbaum Bounds */
Rosenbaum bounds for diff (N = 164 matched pairs)
-----
Gamma      sig+      sig-      t-hat+      t-hat-      CI+      CI-
-----
1          1.1e-16    1.1e-16    2383.35     2383.35     1846.15    2889.71
2          7.9e-07    0          1387.84     3333.11     844.252    3817.07
3          .0010003   0          842.678     3839.1      356.5      4406.52
4          .025238    0          520.882     4206.31    -1.65872   4809.31
5          .134402    0          306.03      4450.29    -285.575   5191.35
6          .31653     0          122.884     4673.35    -476.323   5506.34
7          .560917    0          -11.9081    4861.71    -647.572   5761.19

* gamma - log odds of differential assignment due to unobserved factors
sig+ - upper bound significance level
sig- - lower bound significance level
t-hat+ - upper bound Hodges-Lehmann point estimate
t-hat- - lower bound Hodges-Lehmann point estimate
CI+ - upper bound confidence interval (alpha = .95)
CI- - lower bound confidence interval (alpha = .95)
    
```



```

rbounds diff, gamma(1(17)) /* Rosenbaum Bounds */
Rosenbaum bounds for diff (N = 164 matched pairs)
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2          7.9e-07    0          1387.84     3333.11     844.252    3817.07
3          .0010003   0          842.678     3839.1      356.5      4406.52
4          .025238    0          520.882     4206.31    -1.65872   4809.31
5          .134402    0          306.03      4450.29    -285.575   5191.35
6          .31653     0          122.884     4673.35    -476.323   5506.34
7          .560917    0          -11.9081    4861.71    -647.572   5761.19

* gamma - log odds of differential assignment due to unobserved factors
sig+ - upper bound significance level
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t-hat+ - upper bound Hodges-Lehmann point estimate
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CI+ - upper bound confidence interval (alpha = .95)
CI- - lower bound confidence interval (alpha = .95)
    
```



Gamma	CI+	CI-
1	1846.15	2889.71
2	844.252	3837.07
3	356.5	4406.52
4	-1.65872	4809.31
5	-285.373	5191.33
6	-476.323	5506.34
7	-643.633	5763.39



Method	Effect	L95%CI	U95%CI
T-test	4,498	3,849	5,147
Bivariate Regression	4,498	3,337	5,158
Fully Adjusted Multi Regression	2,215	1,815	2,616
P-value Adjusted Multi Regression	2,206	1,815	2,597
Pscore: 6 vars, NN match, caliper = 0.03	2,405	1,605	3,205
Pscore: 6 vars, NN match, caliper = 0.03, BS=1k	2,393	1,787	2,913
Pscore: 4 vars, NN match, caliper = 0.05	2,618	1,781	3,455
Pscore: 4 vars, NN match, caliper = 0.001	3,231	2,173	4,289
Pscore: 4 vars, NN match, caliper = 0.9	2,618	1,781	3,455
Pscore: 6 vars, Kernel match	2,730	2,068	3,392

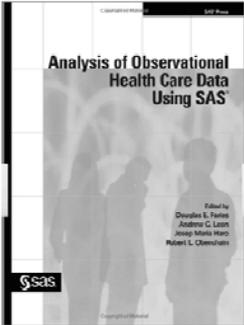
Method	Effect	L95%CI	U95%CI
T-test	4,498	3,849	5,147
Bivariate Regression	4,498	3,337	5,158
Fully Adjusted Multi Regression	2,215	1,815	2,616
P-value Adjusted Multi Regression	2,206	1,745	2,597
Pscore: 6 vars, NN match, caliper = 0.03	2,405	1,727	3,205
Pscore: 6 vars, NN match, caliper = 0.03, BS=1k	2,393	1,787	2,913
Pscore: 4 vars, NN match, caliper = 0.05	2,618	1,781	3,455
Pscore: 4 vars, NN match, caliper = 0.001	3,231	2,173	4,289
Pscore: 4 vars, NN match, caliper = 0.9	2,618	1,781	3,455
Pscore: 6 vars, Kernel match	2,730	2,068	3,392

5. Issues & Assumptions

Image recreated from: Freedman, DA. Oasis or mirage? CHANCE Magazine. 2008; 21: (1):59-61.

- Counterfactuals are unobservable
- Black box mechanisms
- ATE or ATT?
- Pscore matching should balance confounders
- Must assume no unobservables
- Enforce “support”
- Missing values are problematic
- No predictors that are consequence of outcome
- Pcores are mere estimates from one sample

Can also be done in
SAS, R, SPSS,
and other programs



**Analysis of Observational
Health Care Data
Using SAS**

Edited by
Douglas E. Fienberg
Andrew C. Leon
Joseph Maria Mero
Robert L. Glonek

SAS

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For further study, see also

Prof. Liz Stuart's website!

Johns Hopkins University

<http://www.biostat.jhsph.edu/~estuart/propensityscoresoftware.html>

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Simplified Tasks

- (1) Set outcome variable (Y) aside
- (2) Model treatment/exposure (0,1) with logistic regression or perhaps better models
- (3) Check/Assess balance
- (4) Once balance is maximized, use propensity score in analysis to estimate ATT
- (5) Estimate potential impact of unobservables

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Once again... with gusto!

Because they require us to think about the ideal experiment we would have liked to have conducted, propensity score methods are a better tool than multiple regression. Setting aside the outcome variable, Y, until it's time to assess differences between observed outcomes and counterfactual substitutes, is an invaluable addition to the practice of applied research.



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6. Review

<p>I. Review of Core Ideas</p> <ul style="list-style-type: none"> a. Causal inference b. Counterfactuals c. Experimental Ideal <p>II. Effect Measures</p> <ul style="list-style-type: none"> a. ATE = ACE b. ATT = TOT c. Regression estimates in obs study <p>III. Typical Analysis</p> <ul style="list-style-type: none"> a. Bivariate Regression b. Multivariate Regression c. Effect? 	<p>IV. Propensity Score Methods</p> <ul style="list-style-type: none"> a. Stata psmatch2 b. Assess balance c. Estimate ATT d. Enforce support e. Bootstrap f. Rosenbaum bounds <p>V. Assumptions & Issues</p> <ul style="list-style-type: none"> a. Black-box mechanisms b. Unobservables c. Balance & TAM d. Missing values e. Support
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7. Questions

1. What's the difference between ATE and ATT?
2. Why is the assumption that outcomes are independent of the TAM so important?
3. Is Stata the only program that fits propensity score models?
4. What does the "gamma" of Rosenbaum bounds mean?
5. What does "enforcing support" mean?



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References & Resources

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2. Gou S, Fraser MW. Propensity Score Analysis: Statistical Methods and Applications. Thousand Oaks, CA: SAGE Publications; 2010:Fig 5.1.
3. Oakes JM, Forsyth A, Schmitz KH. The effects of neighborhood density and street connectivity on walking behavior: the Twin Cities walking study. *Epidemiology Perspectives and Innovations*. 2007; 13: (4):16.
4. Freedman, DA. Oasis or mirage? *CHANCE Magazine*. 2008; 21: (1): 59-61.